El problema de Josefus

Rafael Isaacs
UIS

Hablemos de Olimpiadas Matemáticas

Junio 2018

Josefus

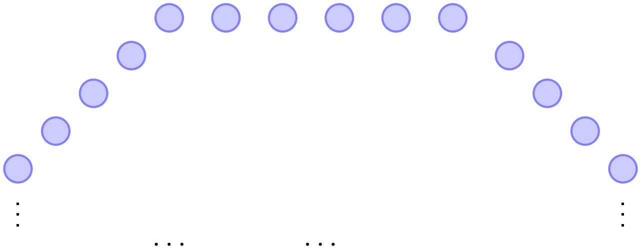
- 70 años después de Cristo.

Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.

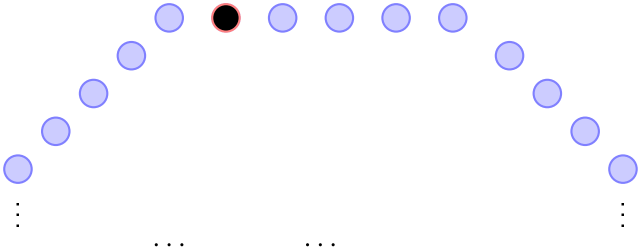
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



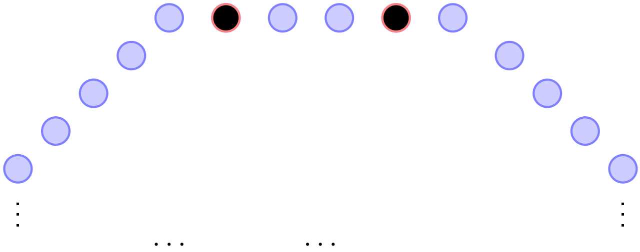
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



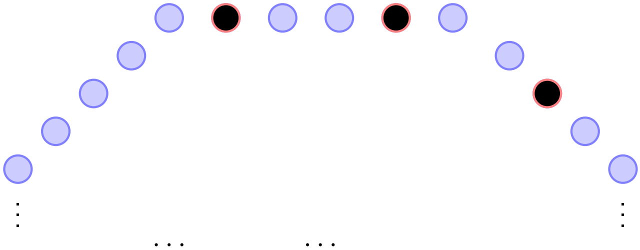
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



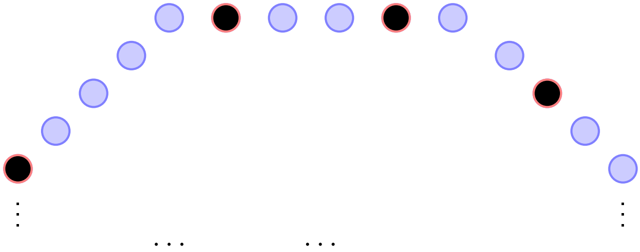
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



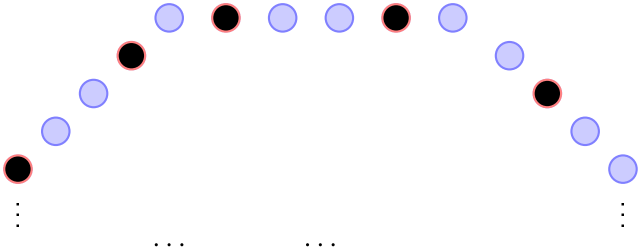
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



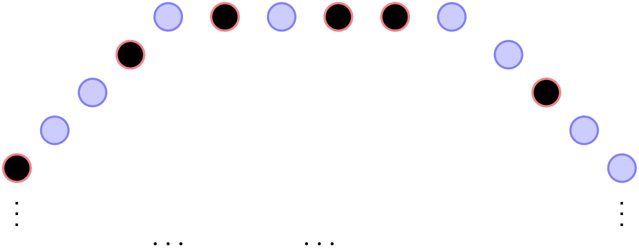
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



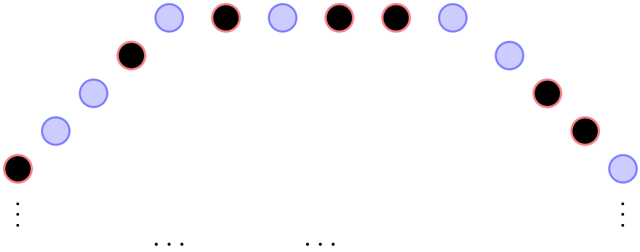
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



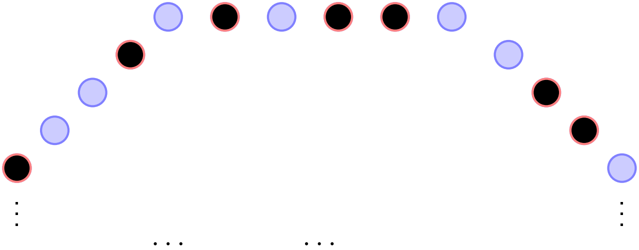
Josefus

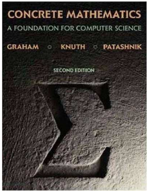
- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.



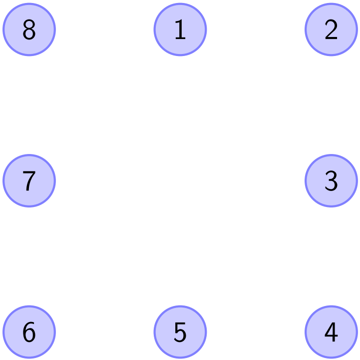
Josefus

- 70 años después de Cristo.
- 41 guerreros deciden su suicidio.

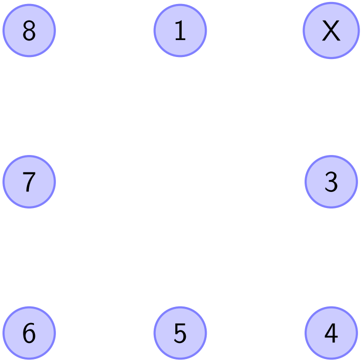




Primeros



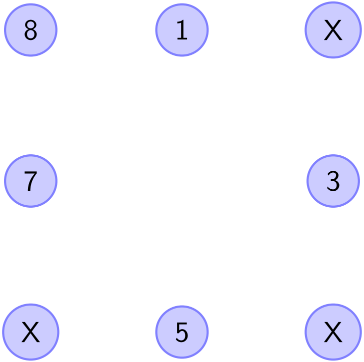
Primeros



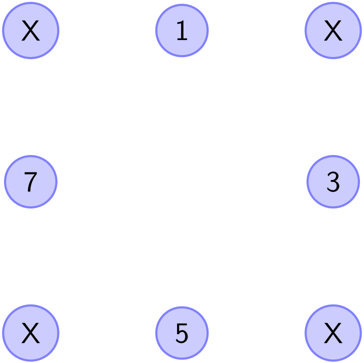
Primeros

8	1	X
7		3
6	5	X

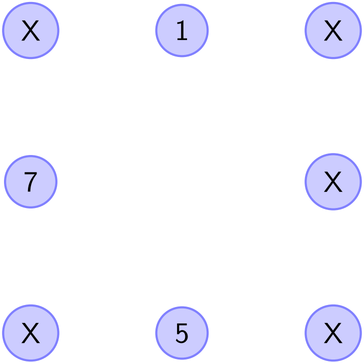
Primeros



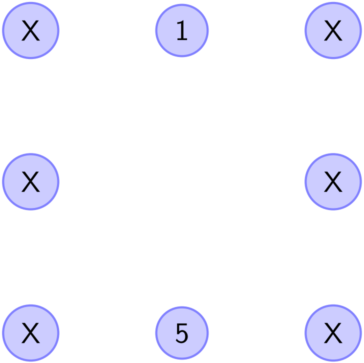
Primeros



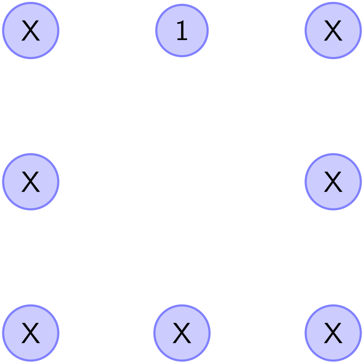
Primeros



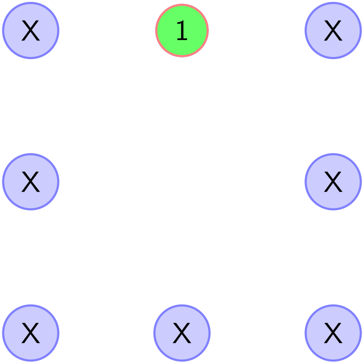
Primeros



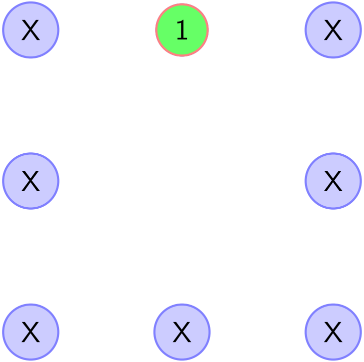
Primeros



Primeros

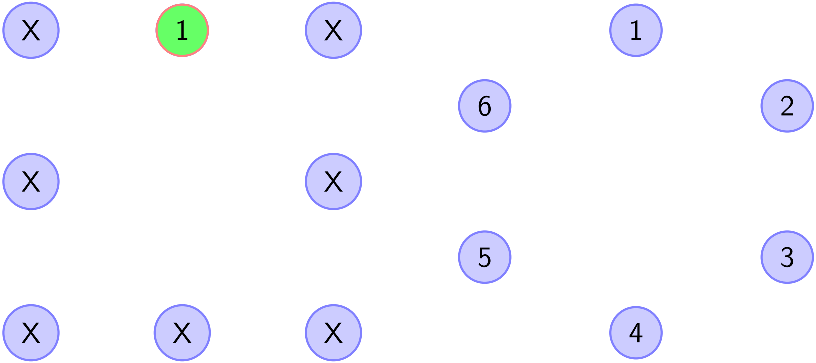


Primeros



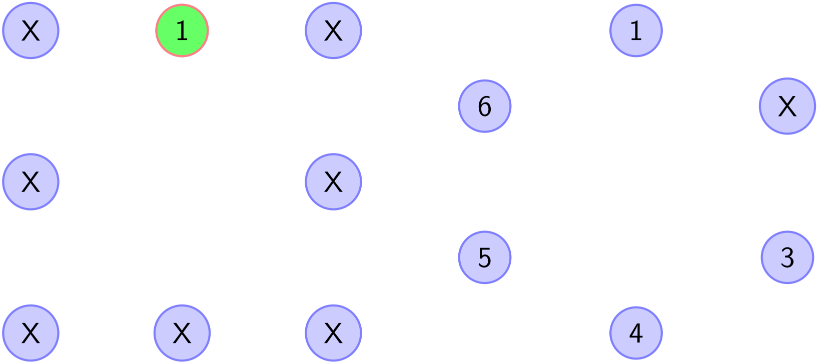
$\boxed{J(8) = 1}$

Primeros



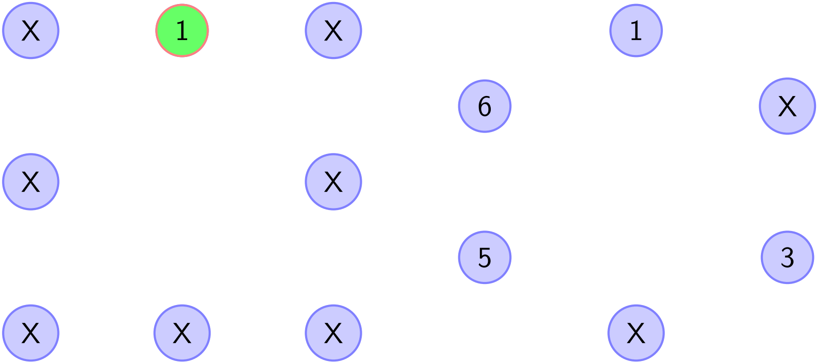
$J(8) = 1$

Primeros



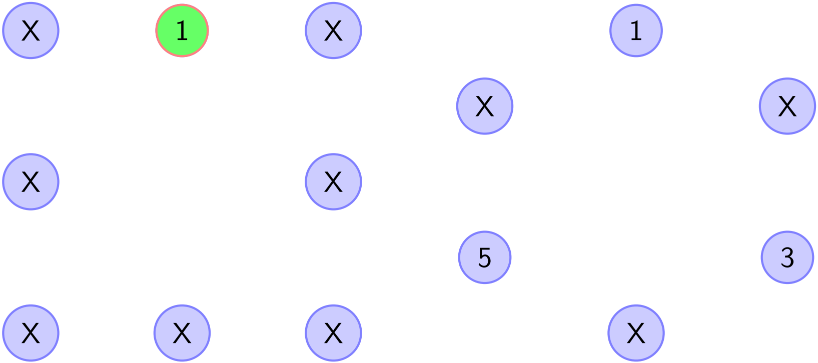
$$\boxed{J(8) = 1}$$

Primeros



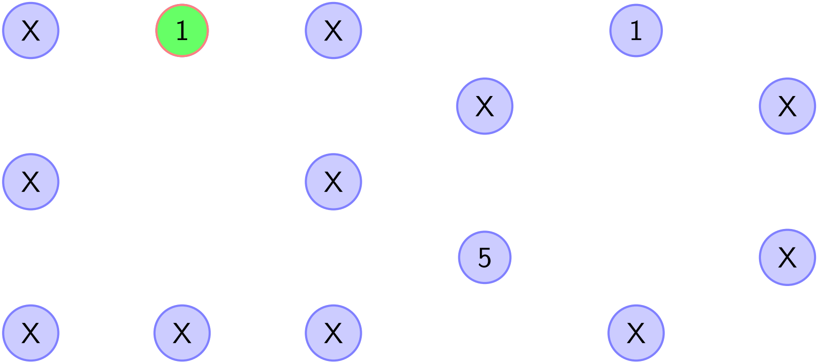
$$\boxed{J(8) = 1}$$

Primeros



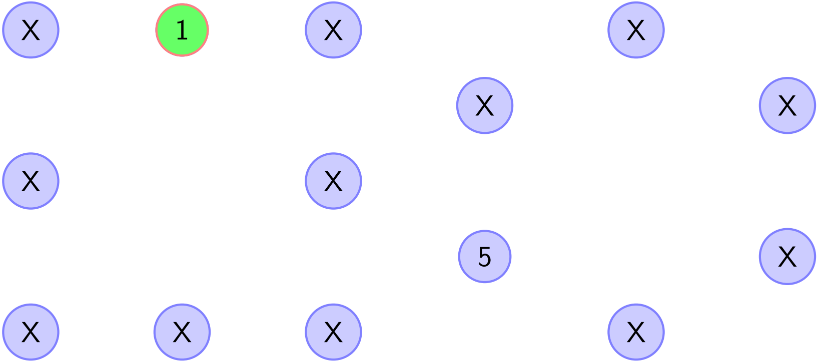
$$\boxed{J(8) = 1}$$

Primeros



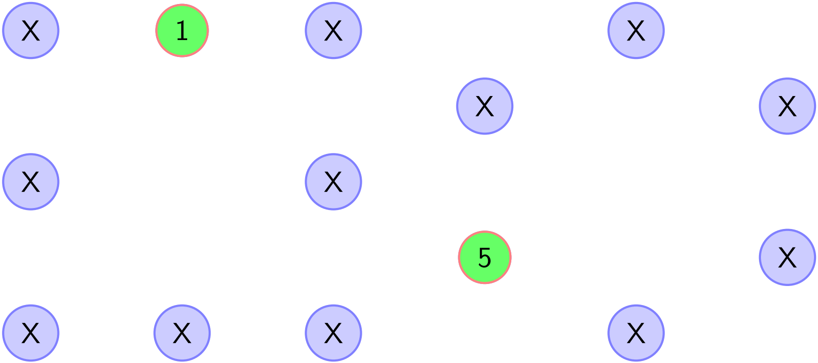
$\boxed{J(8) = 1}$

Primeros



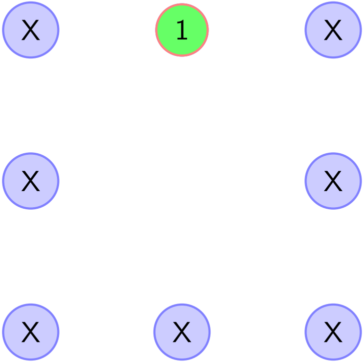
$\boxed{J(8) = 1}$

Primeros

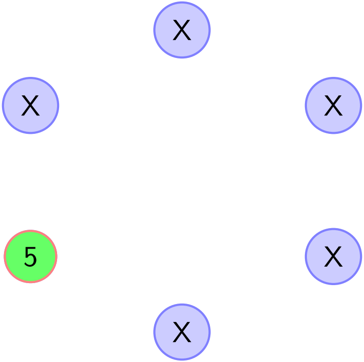


$$\boxed{J(8) = 1}$$

Primeros



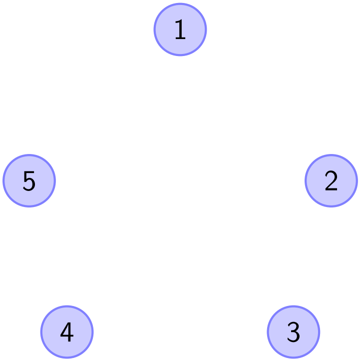
$J(8) = 1$



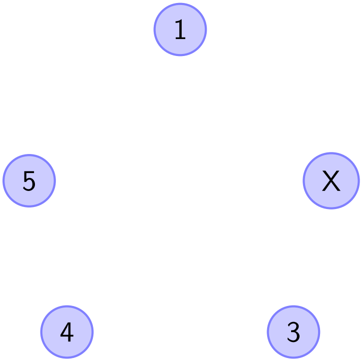
$J(6) = 5$

OTROS

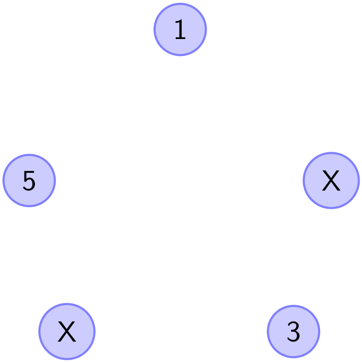
OTROS



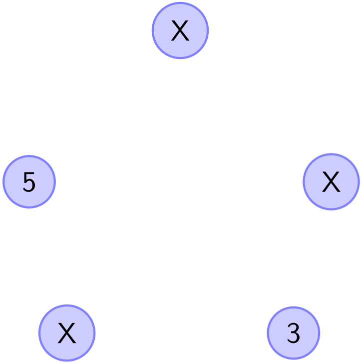
OTROS



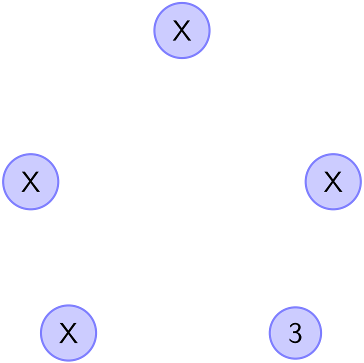
OTROS



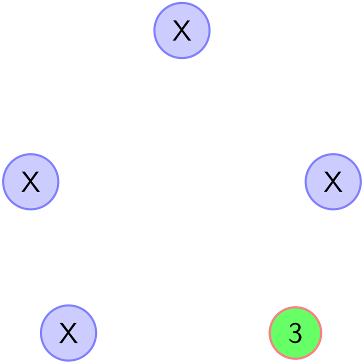
OTROS



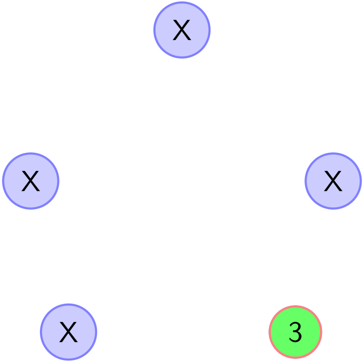
OTROS



OTROS

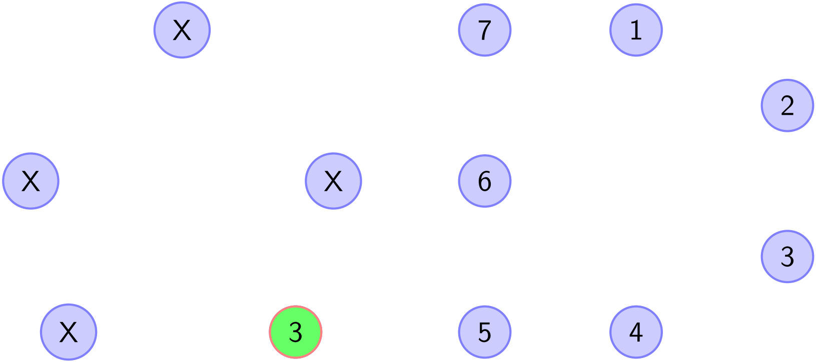


OTROS



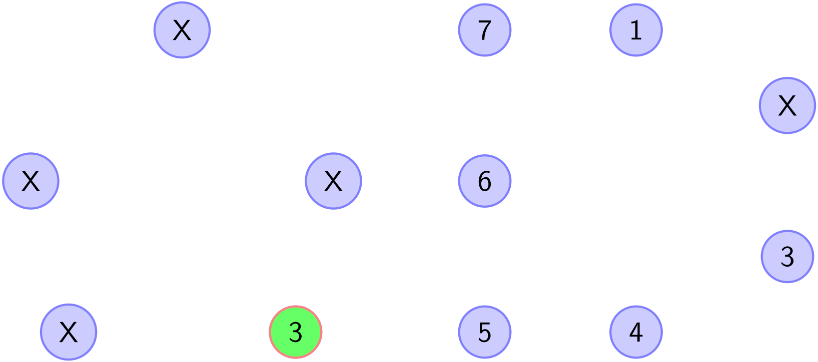
$$J(5) = 3$$

OTROS



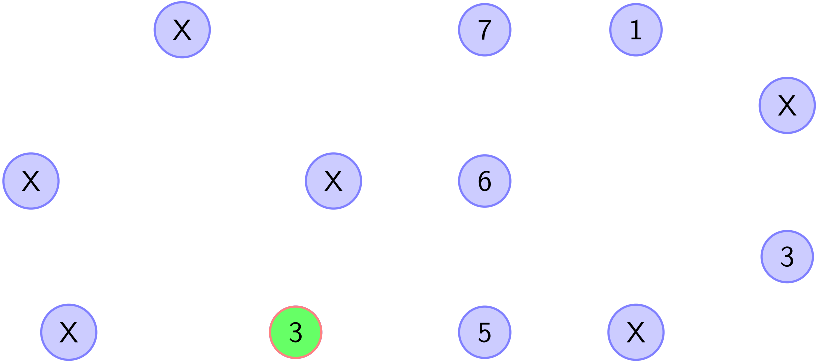
$$\boxed{J(5) = 3}$$

OTROS



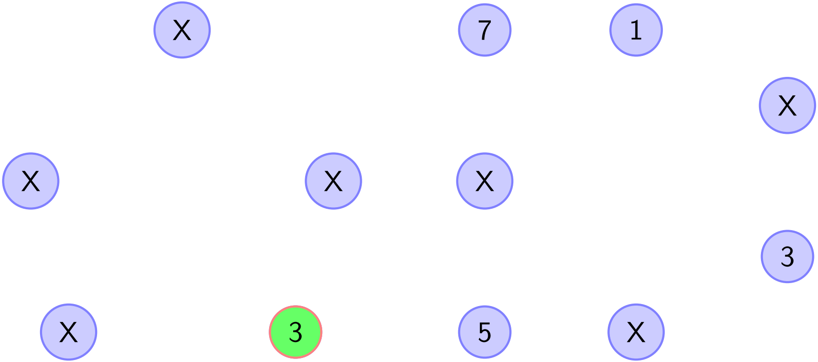
$$\boxed{J(5) = 3}$$

OTROS



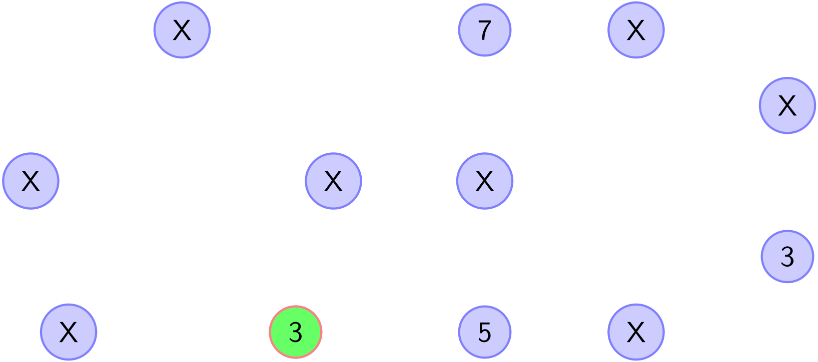
$J(5) = 3$

OTROS



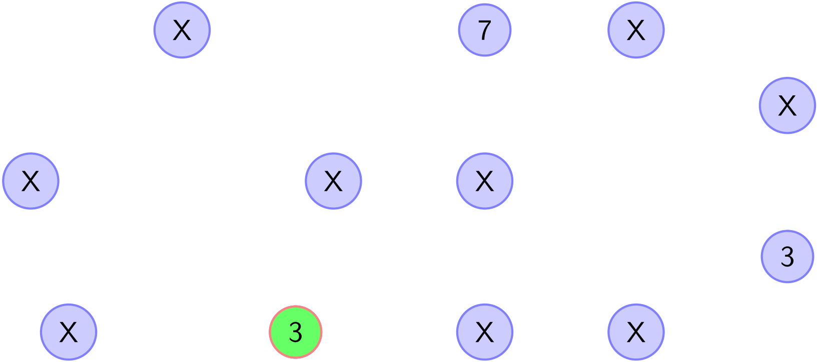
$\boxed{J(5) = 3}$

OTROS



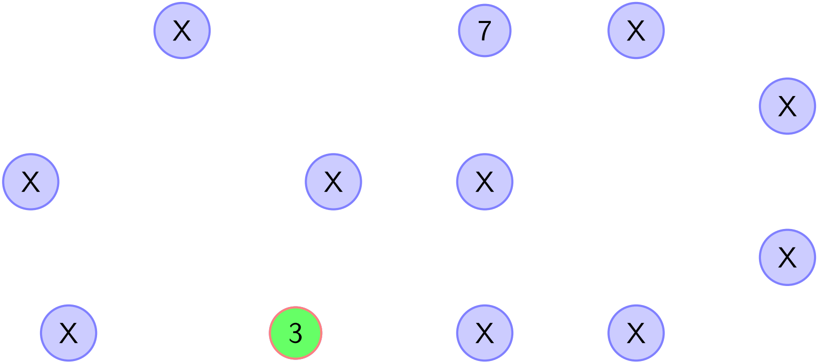
$\boxed{J(5) = 3}$

OTROS



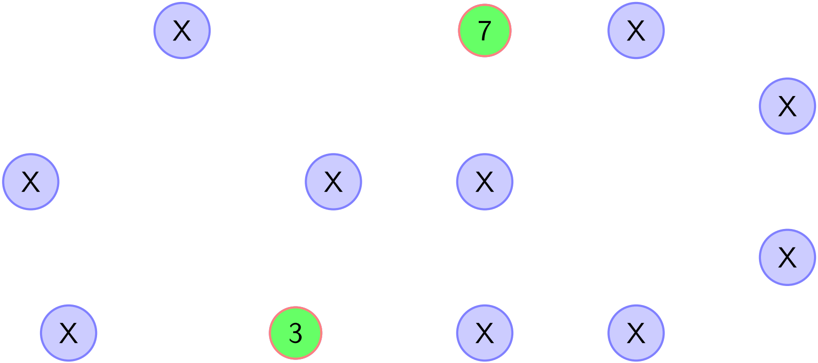
$$\boxed{J(5) = 3}$$

OTROS



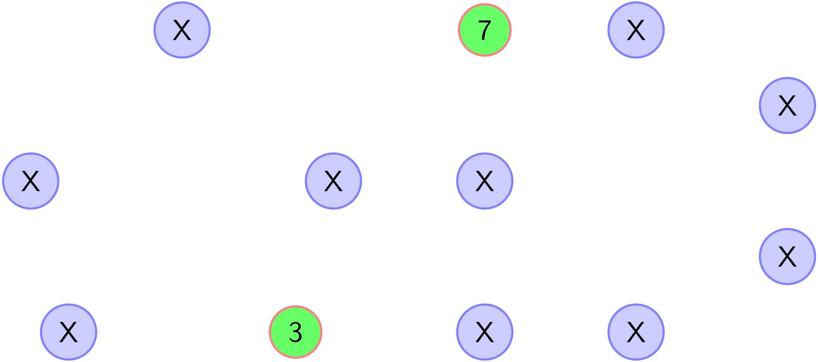
$\boxed{J(5) = 3}$

OTROS



$\boxed{J(5) = 3}$

OTROS



$J(5) = 3$

$J(7) = 7$

¿Qué queremos?

- Una fórmula recurrente para $J(n)$:.

¿Qué queremos?

- Una fórmula recurrente para $J(n)$:.
- Que nos ayude a conjeturar propiedades: puntos fijos, órbitas.

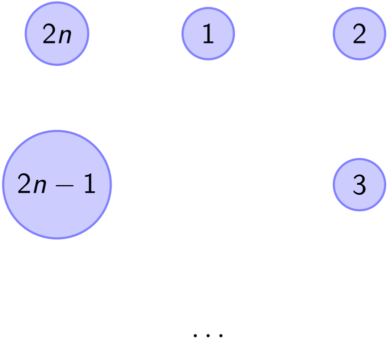
¿Qué queremos?

- Una fórmula recurrente para $J(n)$:.
- Que nos ayude a conjeturar propiedades: puntos fijos, órbitas.
- Que nos conduzca a maneras rápidas de calcular

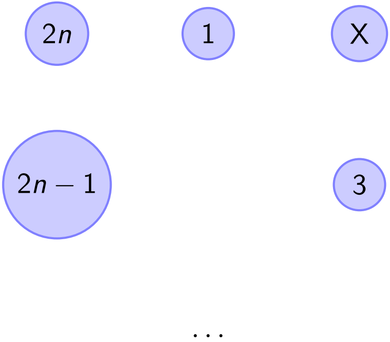
¿Qué queremos?

- Una fórmula recurrente para $J(n)$:.
- Que nos ayude a conjeturar propiedades: puntos fijos, órbitas.
- Que nos conduzca a maneras rápidas de calcular
- Una fórmula explícita que nos demuestre nuestras inquietudes.

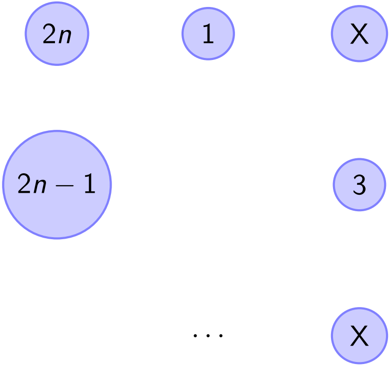
Recurrencia para $2n$



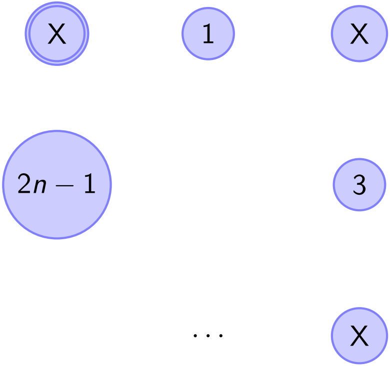
Recurrencia para $2n$



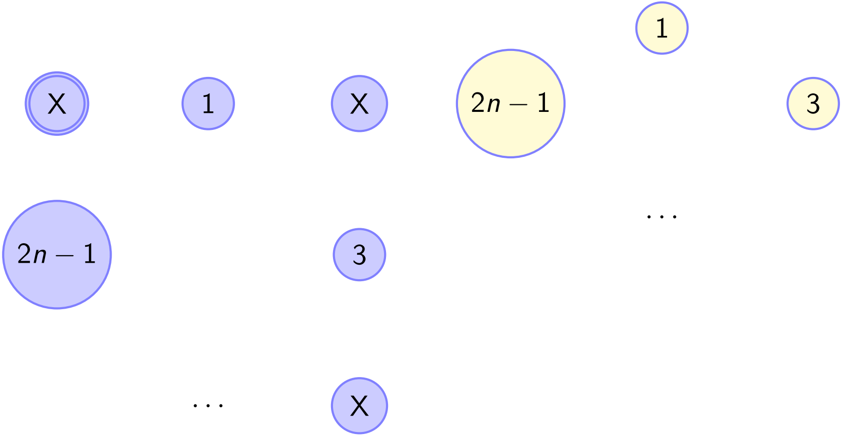
Recurrencia para $2n$



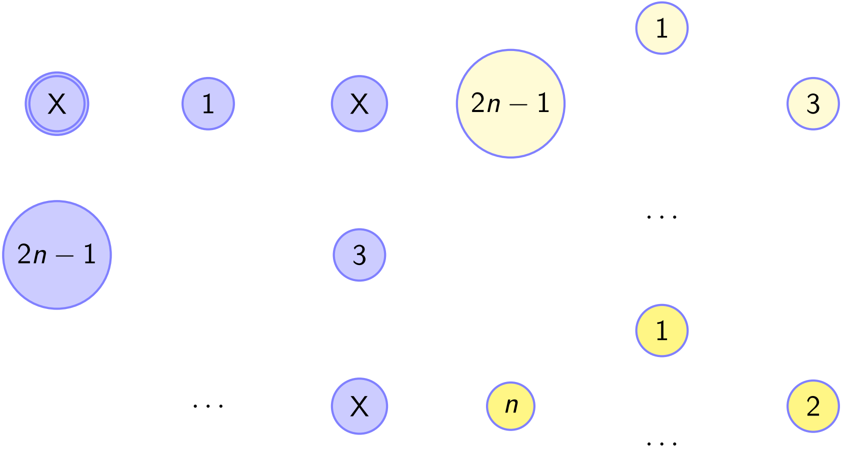
Recurrencia para $2n$



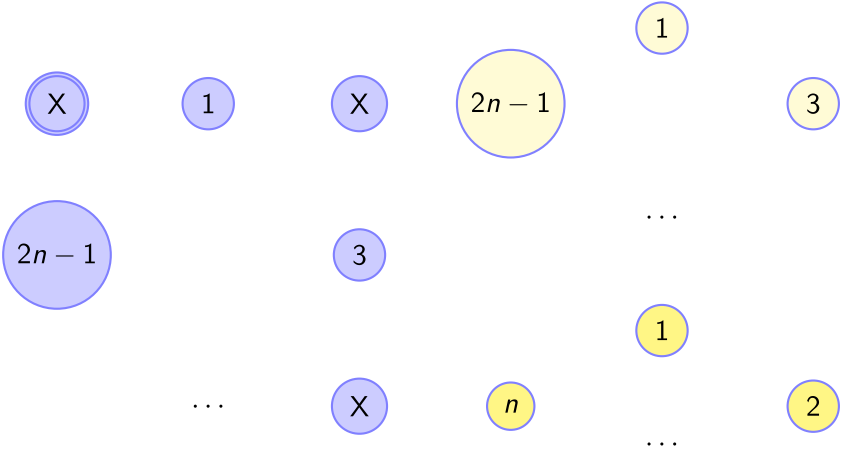
Recurrencia para $2n$



Recurrencia para $2n$

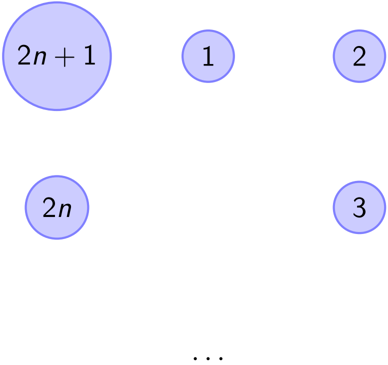


Recurrencia para $2n$

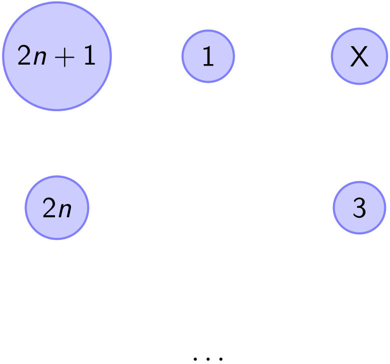


$$\boxed{J(2n) = 2J(n) - 1}$$

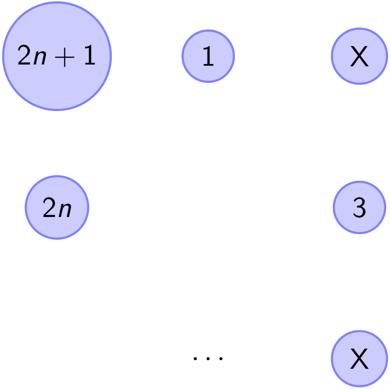
Recurrencia para $2n + 1$



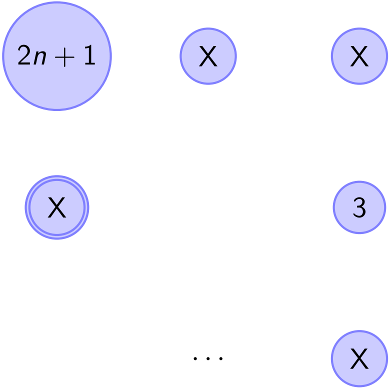
Recurrencia para $2n + 1$



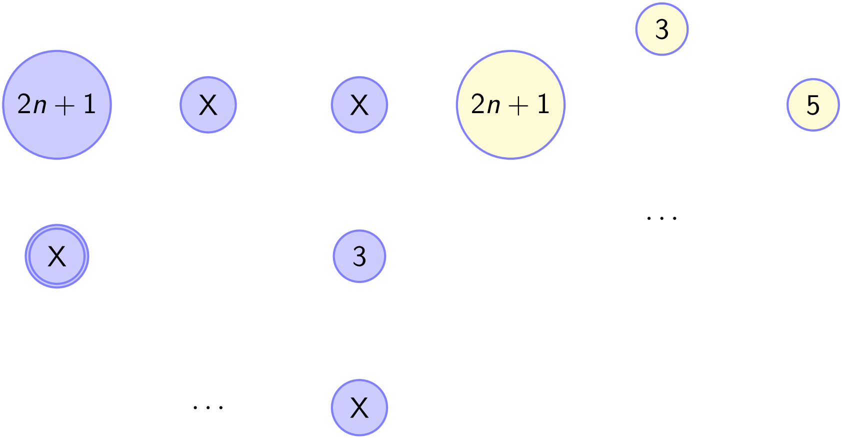
Recurrencia para $2n+1$



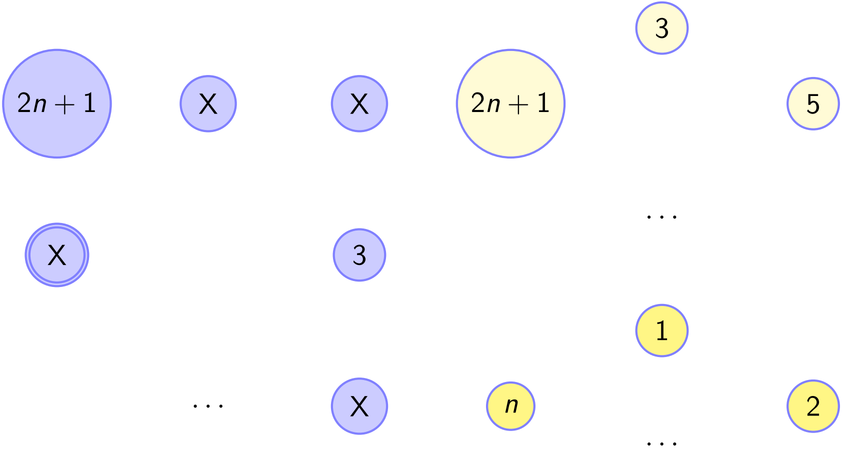
Recurrencia para $2n + 1$



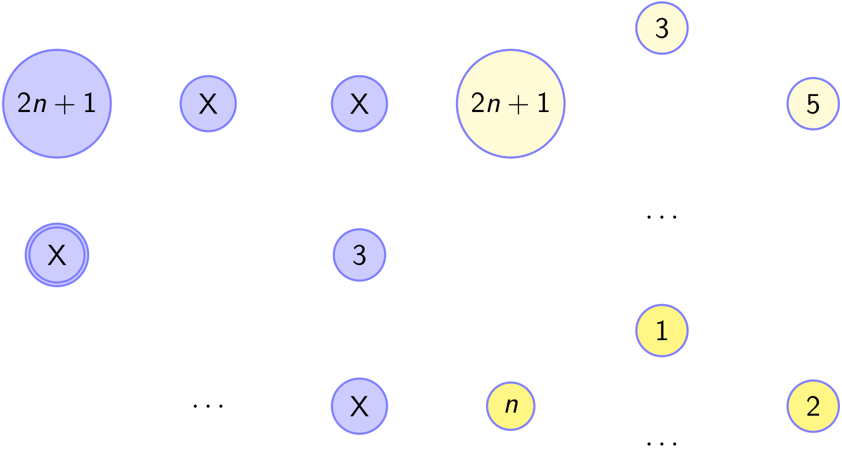
Recurrencia para $2n+1$



Recurrencia para $2n + 1$



Recurrencia para $2n+1$



$$\boxed{J(2n+1) = 2J(n) + 1}$$

Conocemos

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
							1									

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
					5		1									

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
				3	5		1									

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
				3	5	7	1									

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
				3	5	7	1									

$$\boxed{J(2^n) = 1}$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1				3	5	7	1									

$$\boxed{J(2^n) = 1}$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1			3	5	7	1									

$$\boxed{J(2^n) = 1}$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1		1	3	5	7	1									

$$\boxed{J(2^n) = 1}$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1		1	3	5	7	1								1	

$$\boxed{J(2^n) = 1}$$

Conocemos

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1		1	3	5	7	1								1	

$$J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3$$

$$J(2^n) = 1$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1								1	

$$\boxed{J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3}$$

$$\boxed{J(2^n) = 1}$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1								1	

$$\boxed{J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3}$$

$$\boxed{J(9) = 2J(4) + 1 = 2 \times 1 + 1 = 3}$$

$$\boxed{J(2^n) = 1}$$

Conocemos

$$\boxed{J(2n) = 2J(n) - 1}$$

$$\boxed{J(2n+1) = 2J(n) + 1}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3							1	

$$\boxed{J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3}$$

$$\boxed{J(9) = 2J(4) + 1 = 2 \times 1 + 1 = 3}$$

$$\boxed{J(2^n) = 1}$$

Conocemos

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3							1	

$$J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3$$

$$J(9) = 2J(4) + 1 = 2 \times 1 + 1 = 3$$

$$J(2^n) = 1 \qquad J(10) = 2J(5) - 1 = 2 \times 3 - 1 = 5$$

Conocemos

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5						1	

$$J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3$$

$$J(9) = 2J(4) + 1 = 2 \times 1 + 1 = 3$$

$$J(2^n) = 1 \qquad J(10) = 2J(5) - 1 = 2 \times 3 - 1 = 5$$

Conocemos

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5						1	

$$J(2^n) = 1$$

$$J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3$$

$$J(9) = 2J(4) + 1 = 2 \times 1 + 1 = 3$$

$$J(10) = 2J(5) - 1 = 2 \times 3 - 1 = 5$$

$$J(11) = 2J(5) + 1 = 2 \times 3 + 1 = 7$$

Conocemos

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7					1	

$$J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3$$

$$J(9) = 2J(4) + 1 = 2 \times 1 + 1 = 3$$

$$J(2^n) = 1$$

$$J(10) = 2J(5) - 1 = 2 \times 3 - 1 = 5$$

$$J(11) = 2J(5) + 1 = 2 \times 3 + 1 = 7$$

Conocemos

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	

$$J(3) = 2J(1) + 1 = 2 \times 1 + 1 = 3$$

$$J(9) = 2J(4) + 1 = 2 \times 1 + 1 = 3$$

$$J(2^n) = 1$$

$$J(10) = 2J(5) - 1 = 2 \times 3 - 1 = 5$$

$$J(11) = 2J(5) + 1 = 2 \times 3 + 1 = 7$$

Un cálculo diferente

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1																

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3														

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7										

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15		

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Lema

Si $2^m < k < 2^{m+1}$ *entonces* $J(k) - J(k-1) = 2$

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Lema

Si $2^m < k < 2^{m+1}$ *entonces* $J(k) - J(k-1) = 2$

Demostración.

Si $k = 2n + 1$ entonces $k - 1 = 2n$ por tanto

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Lema

Si $2^m < k < 2^{m+1}$ *entonces* $J(k) - J(k-1) = 2$

Demostración.

Si $k = 2n + 1$ entonces $k - 1 = 2n$ por tanto
$J(k) - J(k-1) = (2J(n) + 1) - (2J(n) - 1) = 2$

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Lema

Si $2^m < k < 2^{m+1}$ *entonces* $J(k) - J(k-1) = 2$

Demostración.

Si $k = 2n + 1$ entonces $k - 1 = 2n$ por tanto

$J(k) - J(k-1) = (2J(n) + 1) - (2J(n) - 1) = 2$

Si $k = 2n$ entonces $k - 1 = 2n - 1$ por tanto

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Lema

Si $2^m < k < 2^{m+1}$ *entonces* $J(k) - J(k-1) = 2$

Demostración.

Si $k = 2n + 1$ entonces $k - 1 = 2n$ por tanto
$J(k) - J(k-1) = (2J(n) + 1) - (2J(n) - 1) = 2$
Si $k = 2n$ entonces $k - 1 = 2n - 1$ por tanto $J(k) - J(k-1) =$
$(2J(n) - 1) - (2J(n-1) + 1) = 2(J(n) - (J(n-1)) - 2 =$

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Lema

Si $2^m < k < 2^{m+1}$ *entonces* $J(k) - J(k-1) = 2$

Demostración.

Si $k = 2n + 1$ entonces $k - 1 = 2n$ por tanto
$J(k) - J(k-1) = (2J(n) + 1) - (2J(n) - 1) = 2$
Si $k = 2n$ entonces $k - 1 = 2n - 1$ por tanto $J(k) - J(k-1) =$
$(2J(n) - 1) - (2J(n-1) + 1) = 2(J(n) - (J(n-1)) - 2 = 4 - 2 = 2$

■

Un cálculo diferente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Lema

Si $2^m < k < 2^{m+1}$ *entonces* $J(k) - J(k-1) = 2$

Demostración.

Si $k = 2n+1$ entonces $k-1 = 2n$ por tanto
$J(k) - J(k-1) = (2J(n)+1) - (2J(n)-1) = 2$
Si $k = 2n$ entonces $k-1 = 2n-1$ por tanto $J(k) - J(k-1) =$
$(2J(n)-1) - (2J(n-1)+1) = 2(J(n) - (J(n-1)) - 2 = 4 - 2 = 2$

■

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j+1$.

Cálculo más eficiente

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

$J(11) = J(8+3) =$

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

$J(11) = J(8+3) = 3 \times 2 + 1 = 7$

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

$J(11) = J(8+3) = 3 \times 2 + 1 = 7$
$J(150) = J(128+22) =$

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

$J(11) = J(8+3) = 3 \times 2 + 1 = 7$
$J(150) = J(128+22) = 22 \times 2 + 1 = 45$

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

$J(11) = J(8+3) = 3 \times 2 + 1 = 7$
$J(150) = J(128+22) = 22 \times 2 + 1 = 45$
$J(2^{123} + 1) ==$

Cálculo más eficiente

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

$J(11) = J(8+3) = 3 \times 2 + 1 = 7$
$J(150) = J(128+22) = 22 \times 2 + 1 = 45$
$J(2^{123} + 1) == \ = 3$

Usemos base 2: ejemplo

$150 = 2 \times 75 + 0$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$$

Usemos base 2: ejemplo

$$150 = 2\times 75 + 0 = 2\times(2\times 37+1)+0$$
$$= 2\times(2\times(2\times 18+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times 9+0)+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times(2\times 4+1)+0)+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times(2\times(2\times 2+0)+1)+0)+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times(2\times(2\times(2\times 1+0)+0)+1)+0)+1)+1)+0$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$$
$$= (10010110)_2 =$$

Usemos base 2: ejemplo

$$
\begin{aligned}
150 &= 2\times 75+0 = 2\times(2\times 37+1)+0\\
&= 2\times(2\times(2\times 18+1)+1)+0\\
&= 2\times(2\times(2\times(2\times 9+0)+1)+1)+0\\
&= 2\times(2\times(2\times(2\times(2\times 4+1)+0)+1)+1)+0\\
&= 2\times(2\times(2\times(2\times(2\times(2\times 2+0)+1)+0)+1)+1)+0\\
&= 2\times(2\times(2\times(2\times(2\times(2\times(2\times 1+0)+0)+1)+0)+1)+1)+0\\
&= 2^7+0\times 2^6+0\times 2^5+1\times 2^4+0\times 2^3+1\times 2^2+1\times 2^1+0\\
&= (10010110)_2 = 128+16+4+2
\end{aligned}
$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$$
$$= (10010110)_2 = 128 + 16 + 4 + 2$$
$$= (10000000)_2 + (0010110)_2 =$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$$
$$= (10010110)_2 = 128 + 16 + 4 + 2$$
$$= (10000000)_2 + (0010110)_2 = 2^m + j$$
$$J(2^m + j) = 2j + 1 =$$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$$
$$= (10010110)_2 = 128 + 16 + 4 + 2$$
$$= (10000000)_2 + (0010110)_2 = 2^m + j$$
$$J(2^m + j) = 2j + 1 = 2 \times (0010110)_2 + 1 =$$

Usemos base 2: ejemplo

$$
\begin{aligned}
150 &= 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0 \\
&= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0 \\
&= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0 \\
&= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0 \\
&= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0 \\
&= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0 \\
&= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \\
&= (10010110)_2 = 128 + 16 + 4 + 2 \\
&= (10000000)_2 + (0010110)_2 = 2^m + j
\end{aligned}
$$

$$J(2^m + j) = 2j + 1 = 2 \times (0010110)_2 + 1 = (00101101)_2 =$$

Usemos base 2: ejemplo

$$150 = 2\times 75+0 = 2\times(2\times 37+1)+0$$
$$= 2\times(2\times(2\times 18+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times 9+0)+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times(2\times 4+1)+0)+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times(2\times(2\times 2+0)+1)+0)+1)+1)+0$$
$$= 2\times(2\times(2\times(2\times(2\times(2\times(2\times 1+0)+0)+1)+0)+1)+1)+0$$
$$= 2^7+0\times 2^6+0\times 2^5+1\times 2^4+0\times 2^3+1\times 2^2+1\times 2^1+0$$
$$= (10010110)_2 = 128+16+4+2$$
$$= (10000000)_2+(0010110)_2 = 2^m+j$$
$$J(2^m+j) = 2j+1 = 2\times(0010110)_2+1 = (00101101)_2 =$$
$$32+8+4+1 ==$$

Usemos base 2: ejemplo

$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$
$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$
$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$
$= (10010110)_2 = 128 + 16 + 4 + 2$
$= (10000000)_2 + (0010110)_2 = 2^m + j$
$J(2^m + j) = 2j + 1 = 2 \times (0010110)_2 + 1 = (00101101)_2 =$
$32 + 8 + 4 + 1 == 45$

Usemos base 2: ejemplo

$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$
$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$
$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$
$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$
$= (10010110)_2 = 128 + 16 + 4 + 2$
$= (10000000)_2 + (0010110)_2 = 2^m + j$
$J(2^m + j) = 2j + 1 = 2 \times (0010110)_2 + 1 = (00101101)_2 =$
$32 + 8 + 4 + 1 == 45$
$J((10010110)_2) = (101101)_2$

Usemos base 2: ejemplo

$$150 = 2 \times 75 + 0 = 2 \times (2 \times 37 + 1) + 0$$
$$= 2 \times (2 \times (2 \times 18 + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times 9 + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times 4 + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2 \times (2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 1 + 0) + 0) + 1) + 0) + 1) + 1) + 0$$
$$= 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$$
$$= (10010110)_2 = 128 + 16 + 4 + 2$$
$$= (10000000)_2 + (0010110)_2 = 2^m + j$$
$$J(2^m + j) = 2j + 1 = 2 \times (0010110)_2 + 1 = (00101101)_2 =$$
$$32 + 8 + 4 + 1 == 45$$
$$J((10010110)_2) = (101101)_2$$

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \ldots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10\ldots0)_2$ y

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \ldots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10 \ldots 0)_2$ y $j = (b_{m-1} \ldots b_1 b_0)_2$

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \ldots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10 \ldots 0)_2$ y $j = (b_{m-1} \ldots b_1 b_0)_2$ entonces
$2j = (b_{m-1} \ldots b_1 b_0 0)_2$

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \ldots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10\ldots0)_2$ y $j = (b_{m-1} \ldots b_1 b_0)_2$ entonces
$2j = (b_{m-1} \ldots b_1 b_0 0)_2$ y
$2j + 1 = (b_{m-1} \ldots b_1 b_0 1)_2 =$

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \ldots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10\ldots0)_2$ y $j = (b_{m-1} \ldots b_1 b_0)_2$ entonces
$2j = (b_{m-1} \ldots b_1 b_0 0)_2$ y
$2j + 1 = (b_{m-1} \ldots b_1 b_0 1)_2 = (b_{m-1} \ldots b_1 b_0 b_m)_2 = J(k)$

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \dots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10 \dots 0)_2$ y $j = (b_{m-1} \dots b_1 b_0)_2$ entonces
$2j = (b_{m-1} \dots b_1 b_0 0)_2$ y
$2j + 1 = (b_{m-1} \dots b_1 b_0 1)_2 = (b_{m-1} \dots b_1 b_0 b_m)_2 = J(k)$ es decir
$J(k) = J(b_m b_{m-1} \dots b_1 b_0)_2 =$

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$*.*

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \dots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10\dots0)_2$ y $j = (b_{m-1} \dots b_1 b_0)_2$ entonces
$2j = (b_{m-1} \dots b_1 b_0 0)_2$ y
$2j + 1 = (b_{m-1} \dots b_1 b_0 1)_2 = (b_{m-1} \dots b_1 b_0 b_m)_2 = J(k)$ es decir
$J(k) = J(b_m b_{m-1} \dots b_1 b_0)_2 = (b_{m-1} \dots b_1 b_0 b_m)_2$ con $b_m = 1$,

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ *con* $0 \leq j < 2^m$ *entonces* $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \dots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10\dots0)_2$ y $j = (b_{m-1} \dots b_1 b_0)_2$ entonces
$2j = (b_{m-1} \dots b_1 b_0 0)_2$ y
$2j + 1 = (b_{m-1} \dots b_1 b_0 1)_2 = (b_{m-1} \dots b_1 b_0 b_m)_2 = J(k)$ es decir
$J(k) = J(b_m b_{m-1} \dots b_1 b_0)_2 = (b_{m-1} \dots b_1 b_0 b_m)_2$ con $b_m = 1$,
$J(41) = J(101001)_2 =$

Usemos base 2: general

Tenemos

Proposición

Si $k = 2^m + j$ con $0 \leq j < 2^m$ entonces $J(k) = 2j + 1$.

Si $k = 2^m + j$ entonces $k = (b_m b_{m-1} \ldots b_1 b_0)_2$ con $b_m = 1$,
Así $2^m = (10 \ldots 0)_2$ y $j = (b_{m-1} \ldots b_1 b_0)_2$ entonces
$2j = (b_{m-1} \ldots b_1 b_0 0)_2$ y
$2j + 1 = (b_{m-1} \ldots b_1 b_0 1)_2 = (b_{m-1} \ldots b_1 b_0 b_m)_2 = J(k)$ es decir
$J(k) = J(b_m b_{m-1} \ldots b_1 b_0)_2 = (b_{m-1} \ldots b_1 b_0 b_m)_2$ con $b_m = 1$,
$J(41) = J(101001)_2 = (010011)_2 = 19$.

Otras

1. Los puntos fijos.

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 =$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2)$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2) \Leftrightarrow 3|2^m - 2$$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2) \Leftrightarrow 3|2^m - 2$$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2) \Leftrightarrow 3|2^m - 2$

$\boxed{m}$ $\boxed{j}$ $\boxed{k = 2^m + j}$ $\boxed{J(k) = 2j + 1}$ $\boxed{\text{bin}}$

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2) \Leftrightarrow 3|2^m - 2$$

m	j	$k = 2^m + j$	$J(k) = 2j + 1$	bin
1	0	2	2	10

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2) \Leftrightarrow 3 | 2^m - 2$

m	j	$k = 2^m + j$	$J(k) = 2j + 1$	bin
1	0	2	2	10
3	2	10	5	1010

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2) \Leftrightarrow 3 | 2^m - 2$$

m	j	$k = 2^m + j$	$J(k) = 2j + 1$	bin
1	0	2	2	10
3	2	10	5	1010
5	10	42	21	101010

Otras

1. Los puntos fijos. $J(k) = k \Leftrightarrow k = 2^m - 1$
2. Las órbitas
   $J^{(8)}(23403) = J(101101101101011)_2 = 2^{10} - 1 = 1023$
3. $J(k) = k/2$

$$J(k) = k/2 \Leftrightarrow 2j + 1 = (2^m + j)/2 \Leftrightarrow j = \frac{1}{3}(2^m - 2) \Leftrightarrow 3|2^m - 2$$

m	j	$k = 2^m + j$	$J(k) = 2j + 1$	bin
1	0	2	2	10
3	2	10	5	1010
5	10	42	21	101010
7	42	170	85	10101010