SURFACE ROUGHNESS ESTIMATION USING FRACTAL VARIOGRAM ANALYSIS

D. Rees & J-P.Muller
Department of Photogrammetry and Surveying,
University College London,
Gower Street,
London WC1E 6BT.
INTERNET: drees@ps.ucl.ac.uk

Fractal dimension is a potentially valuable means of quantifying the roughness of an entire Digital Elevation Model, and/or its sub-regions. However, it has found few applications, due to the fact that it is computationally expensive, and that the intervals of constant fractal dimension had to be determined manually.

A technique has been developed whereby linear sections within a variogram, and hence scale ranges of constant fractal dimension, may be determined automatically.

A study is being conducted to determine the usefulness of fractal dimension as a tool for classification of terrain types by their associated quantified roughness. Variograms for grids of small tiles covering the DEM have been obtained, from which linear intervals, and hence fractal dimension, have been automatically determined.

The method has been applied to a stereo matched SPOT DEM, and to the USGS USA 30 second DEM. The DEMs have been segmented, and comparisons of fractal behaviour with othe measures of the terrain are described.

Keywords: DEM Segmentation, Fractal Dimension, Variogram, Surface Roughness.

INTRODUCTION

Fractal techniques have gradually gained a reputation as a way of rendering visually realistic terrain. In particular, much attention has been focused on fractional Brownian motion ([2]; [6] ; [7]). Despite the fact that this process has not been derived from models of terrain formation, it still persists as a useful application of fractal geometry to terrain datasets. Fractals are basically spatial distributions or patterns which possess self-similarity so that there exists a statistical equivalence between small-scale and large-scale fluctuations in these patterns. Many patterns observed in real world data (point distributions, curves and surfaces) appear to be of self-similar fractal form, such as coastlines which appear to be similar at different scales.

Fractals are characterised by their fractional dimension D, which gives a measure of the change in the properties of a phenomena with scale. In a pure mathematical model, fractal curves maintain these properties over infinite ranges of scale, though in real data, finite limits have been observed. Thus, fractal models of real phenomena include inner and outer cutoff scales, which determine the limits within which similar scaling behaviour is dominant. More than one interval can exist within a wide scale range, so discrete behaviour changes over large scale changes have been recorded.

Studies by Richardson, interpreted in [4], detail the scaling of coastlines by a constant exponent, the fractal dimension D, over ranges of scale of up to two orders of magnitude. A different model of terrain, fractional Brownian motion, was applied to contour data in [3]. The model was further studied in [8], which used the variogram of elevation differences between points a known distance apart. Data sampled using the GESTALT photomapper from United States Geological Survey 30 metre Digital Elevation Models (DEMs) was used.

Breaks in the linearity of the graph were detected visually. All but one of the variograms exhibited linear behaviour over limited ranges of scale (0.6 to 5 Km). Some exhibited distinct linear behaviour of several ranges of scale within the same variogram.

Our aim is to automate this process, such that regional variation of fractal dimension may be detected. This is only feasible if variograms are computed for arbitrarily many portions of the DEM, and from this, fractal dimension is determined in an unsupervised manner.

Applications of these segmented DEMs include (a) data compression of regional and global data-sets [9]; (b) simulation of sub-pixel scattering effects [10]; (c) estimation of kriging interpolation local functions [11]; (d) surface roughness estimates for climatic models [14].

COMPUTATION OF FRACTAL DIMENSION

The application of fractional Brownian motion to terrain originated from [5]. Fractal behaviour is determined by defining a function applicable to a phenomena, where the function exhibits 'invariant' behaviour over a range of scales. When applied to two dimensional functions of natural stochastic systems, in this case height within a coordinate system, a variety of functions may potentially be used to determine fractal properties.

Our method of fractal dimension computation for DEMs is derived from [8], which uses a variant on Brownian motion known as fractional Brownian motion. As it is only valid to determine the fractal dimension over a scale range of constant dimension value, a linear segmentation algorithm (but not the fractal measurement algorithm) due to [15] and [16] is used to determine such intervals over log-log plots.

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Formally, a random function $Z_H(x)$ exhibits pure fractional Brownian motion if for all x and Δx:

$$Pr[\mid \Delta Z_{\Delta x} \mid \| \Delta x \|^{-H} < y] = erf(y) \qquad (1)$$

where "Pr" denotes probability, H is the dimension of the Brown zeroset (which is trivially related to the fractal dimension), and $\mid \Delta Z_{\Delta x} \mid$ denotes $\mid Z_H(x + \Delta x) - Z_H(x) \mid$.

This function exhibits behaviour that gives a similar shaped distribution at different scales. For fractional Brownian motion, $\Delta Z_{\Delta x}$ are the increments of a single valued function $Z_H(x)$, the increments having a Gaussian distribution and a variance as equation 2.

$$< [\Delta Z_{\Delta x}]^2 > \propto \| \Delta x \|^{2H} \qquad (2)$$

where $<>$ denote averages over many samples of $Z_H(x)$, H is a scaling parameter with range $0 < H < 1$, and Δx is a vector change in position in Euclidean space ($\Re^E, E > 1$).

The power law relation between $\Delta Z_{\Delta x}$ and Δx should be invariant for all Δx for single fractals (which have constant H at all space scales) implying that the shape of the distributions for fixed values of Δx will remain the same (Gaussian). Thus, computation of height change statistics at various space scales will enable H to be derived. Taking logs of equation 2 we have:

$$\ln(< [\Delta Z_{\Delta x}]^2 >) \propto \ln(\| \Delta x \|^{2H})$$

and finally, rearranging the above to give:

$$\ln(< [\Delta Z_{\Delta x}]^2 >) \propto 2H \ln(\| \Delta x \|) \qquad (3)$$

Equation 3 gives a straight line relationship between $\ln(< [\Delta Z_{\Delta x}]^2 >)$ and $\ln \| \Delta x \|$ with gradient $2H$ for a true fractal. Empirical studies in the past have found that fractal components of natural scenes appear to preserve their fractal dimension (and hence H value) over a variety of ranges in Δx. An algorithm to determine these ranges is given later.

H is determined by computing the gradient of least squares regressions of the graph $\ln(< [\Delta Z_{\Delta x}]^2 >)$ against $\ln(\| \Delta x \|)$ for a linear interval.

Finally, the fractal dimension, D, is then obtained from H using the simple relation

$$D = 3 - H \qquad (4)$$

LINEAR SEGMENTATION OF THE VARIOGRAM

A measure of linearity originally put forward by [15] and [16] is defined as:

$$I = \frac{\sqrt{4\mu_{11}^2 + (\mu_{20} - \mu_{02})^2}}{\mu_{20} + \mu_{02}} \qquad (5)$$

where $\mu_{ij}(0 \leq i, j \leq 2; i + j = 2)$ represents the central-order second moment of a set of points in a plane.

Let S_n denote a set of n points (where $n \geq 3$) in a plane of fractal plots as

$$S_n = \{p(\| \Delta x \|)\}_{\| \Delta x \| = \| \Delta x \|_{min}}^{\| \Delta x \|_{min} + n - 1}$$

and I_n represents a measure of linearity computed for S_n. The largest limit $\| \Delta x \|_{max}$ of scale within which the function is linear is determined by:

$$\| \Delta x \|_{max} = \| \Delta x \|_{min} + n^* - 1$$

where n^* is given by

$$n^* = Min \{n \mid I_{n-1} \leq I_n, I_n > I_{n+1}; n \geq 4\} \qquad (6)$$

Note: this is not the equation as given in [15] and [16], which contained the erroneous limits:

$$I_{n-1} < I_n \text{ , } I_n > I_{n+1}; \qquad (7)$$

Given a graph with two perfect sections of linearity, I will gain a measure of 1.0 for the first line. However, when the second line is encountered, I_{n+1} will drop in value, but both I_{n-1} and I_n will remain equal. The test will not detect this as a break in the linearity. Beyond the break in linearity, I will steadily decrease, repeatedly failing the test. Thus, the two perfect linear intervals will be treated as one imperfect linear interval by the original criteria.

In figure 1 a variogram of a DEM shows distinct linear intervals. To determine the extent of the first linear interval, $\| \Delta x \|_{max}$ must be found. The first linear interval of the variogram is shown in figure 1.

Once a linear interval is determined, equation 3 is used as the basis of a least squares fit algorithm, to determine attributes such as the fractal dimension of the interval.

APPLICATION TO DEMS

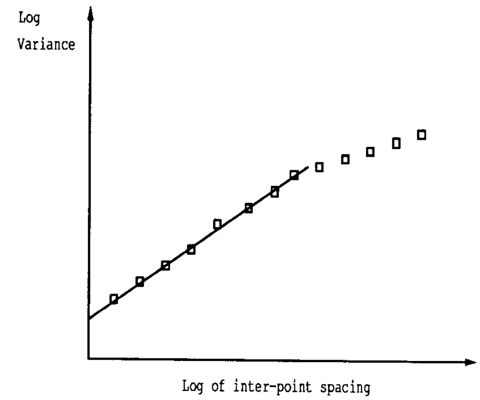
In order to apply segmentation to complete DEMs, fractal dimension must be calculated locally for the entire DEM. To achieve this, we divide the DEM into a matrix of equal sized tiles. These tiles are then treated as complete DEMs within their own right, with a variogram being produced for each.

We have used a tile size of at least 32 by 32 DEM grid elements, giving in the order of a thousand comparisons for each inter-point spacing required. To derive the variance data, all points are compared with all others for small tiles. A larger sample justifies use of a row column algorithm (see [12]), which speeds up computation.

Two DEMs were considered, to give an idea of how the technique performed at different scales. The larger of the two covered a area of 9 363 132 square kilometres, resulting in the use of spherical coordinates to analyse the data. In order to gain an equivalent comparison over the entire DEM, variances were binned at kilometre intervals. In addition, a maximum point pair spacing was guaranteed for the entire DEM, such that all linear intervals were limited in the range of scales used.

Producing graphs and finding linear intervals by hand is an impossible task for this volume of data. Hence, the

Figure 1: Example variogram with first linear interval shown



method of automatic linear segmentation outlined above was used. From theory, we expect fractal dimension to remain constant over a significant range of scales. When the inter-point spacing is plotted against expected absolute difference on a log-log graph, equation 3 implies that a linear interval with gradient H will result.

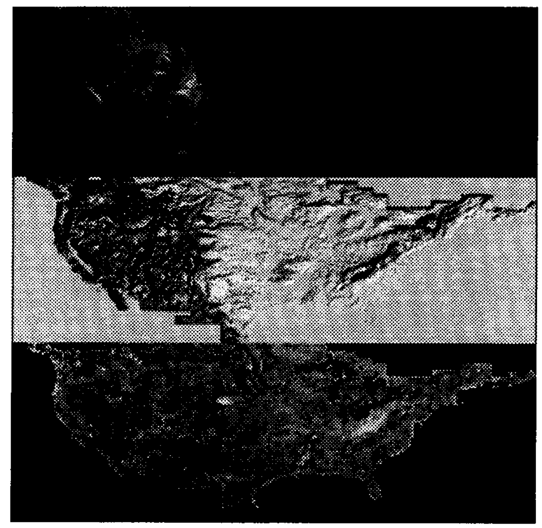
However, when using the linear segmentation method, we may encounter two or three distinct linear intervals for the tile size given above. This leads to the problem of which interval should be chosen to represent the fractal dimension for that particular area. In other studies, where the method has been applied to more local terrain datasets, the first linear interval has been found to give the most consistent results. It avoids the complexity of comparing intervals which do not represent exactly the same scale range. Moreover, the method allows for potential interpolation of the data, as output of the first linear interval can be fed directly into a fractal interpolation program, as applied in [1] and [13].

RESULTS

To observe any relationships between terrain, and its local fractal dimension over a broad area, the USA 30 second DEM was segmented into 40 by 40 tiles, for each of which a variogram was calculated. Tiles were not overlapped, as the resulting fractal dimension was of sufficient detail such that trends could be observed. From these variograms, the first linear interval was detected, and used as the basis for calculating the fractal dimension of the local region. The DEM consists of 8400 by 3360 height samples, giving a resultant tiled image with a resolution of 210 by 84. However, not all points represent an area within the land mass of the United States, so such tiles are given a null fractal dimension value. Figure 2 shows the resulting image.

In order to examine more local effects, the method was applied to a stereo matched SPOT-DEM of Montaigne Sainte Victoire, near Aix-en-Provence [11]. This consisted of 415 by 208 height samples, equi-spaced at 30 metre intervals. This was segmented into 32 by 32 tiles. Figure 3 shows the results. In this figure, overlapping tiles with a grid pitch of 8 samples are used to give a better visual impression of

Figure 2: USGS USA 30 Second DEM images: Top - Height intensity, Mid - Lambertian shaded, Lower - First interval fractal dimension (range -24.97 to 4.39, values in range 2.0 to 3.0 displayed)



the changing fractal dimension. However, for all analysis, non-overlapping tiles were used, such that redundant information was not processed, and time constaints were served.

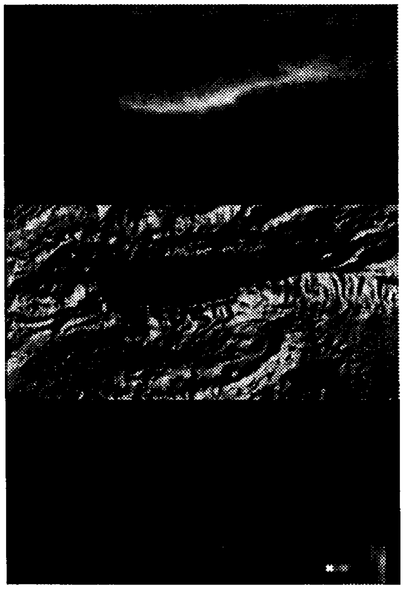
As can be seen, there is some limited correlation between the features in the DEM and the resulting fractal values. To test this further, we investigated the relationships that exist between fractal dimension and rudimentary terrain characteristics. To this end, values resulting from tiled images (40 by 40 sized tiles, non-overlapping) of fractal dimension of the dataset were compared with images, degraded to the same resolution, of height, slope and aspect. The correlation coefficients of fractal dimension with each of the above images were calculated.

For Montaigne Saint Victoire, correlation of dimension against height resulted in a value of 0.44. For dimension against slope, a much higher value of 0.62 was noted. Both the above are extremely significant, with $P < 0.01$, whereas for the correlation of dimension against aspect, a value of 0.01, was observed, giving effectively no correlation at all.

For the USGS USA 30 second DEM, different behaviour was recorded. The correlation of dimension against height gave the best result, a value of 0.45, and a significance level $P < 0.01$. For the other two comparisons, negligable values resulted, with significance level $P > 0.1$. For dimension against aspect, a correlation coeficient of 0.03 was recorded, and for dimension against slope, an even lower value of 0.01 resulted.

A most surprising result was that of the range of fractal dimension values encountered. For Montaigne Sainte Victoire, a range of 2.05 to 2.87 was recorded. However, for the USA DEM, the minimum value measured was -24.97, whereas the maximum value measured was 4.39. Measure-

Figure 1: Montaigne Sainte Victoire stereo-matched DEM images: Top - Height intensity, Middle - Lambertian shaded, Bottom - First interval fractal dimension (range 2.05 to 2.87).



ment of the areas producing these values using 60 by 60 sized tiles yielded similar results. However, 90 percent of values did lie within the range $2 < D < 3$.

Using our row/column algorithm, a time complexity of $O(N^{3/2})$ results, an improvement on the $O(N^2)$ behaviour of the standard method. On a Sun 4/60, calculation of the variance values for the USA dataset occupied 180 CPU minutes. In addition, calculation of the linear segments, and the resulting fractal dimensions took a further 2 CPU minutes.

CONCLUSIONS

Given the size of the dataset under study, the time of three hours taken to aquire the input information for segmentation seems acceptable.

Correlation between fractal dimension and aspect gave negligible results for both samples. However, correlation with slope gave a marked result for the Sainte Victoire DEM, and negligable correlation for the USA. This is probably due to the tile size being larger than most features exhibited by the USA DEM, averaging over any slopes. For Sainte Victoire, a tile may occupy a small proportion of a slope on the central ridge, resulting in meaningful average slope for the whole tile.

Correlation between fractal dimension and height gave an intuitively correct result. It is to be expected that 'rougher' surfaces would occur more commonly in mountainous regions. However, the correlation is not absolute, suggesting that fractal dimension has a role in segmentation that cannot be supplanted by height alone.

The most surprising result of the study is that of fractal dimension exceeding the expected range of $2 < D < 3$. Such values have now been obtained using different algorithms, and different datasets. We are at present determining conditions under which these values can occur.

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