A COMPARISON OF FRACTAL DIMENSION ALGORITHMS USING SYNTHETIC AND EXPERIMENTAL DATA

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ABSTRACT

The fractal dimension (FD) of a waveform represents a powerful tool for transient detection. In particular, in analysis of electroencephalograms (EEG) and electrocardiograms (ECG), this feature has been used to identify and distinguish specific states of physiologic function. A variety of algorithms are available for the computation of FD. In this study, the most common methods of estimating the FD of biomedical signals are analyzed and compared. The analysis is performed over both synthetic data and intracranial EEG (IEEG) data recorded during pre-surgical evaluation of individuals with epileptic seizures. The advantages and drawbacks of each technique are highlighted. The effects of window size, number of overlapping points, and signal to noise ratio (SNR) are evaluated for each method. This study demonstrates that a careful selection of FD algorithm is required for specific applications.

INTRODUCTION

The term "fractal dimension" refers to a non-integer or fractional dimension of any object. FD analysis is frequently used in biomedical signal processing, including EEG analysis [1]-[5]. Applications of FD in this setting include both a temporal approach, that estimates the FD of a waveform, and phase space analysis, which estimates the FD of an attractor. Calculating the FD of waveforms is useful for transient detection, with the additional advantage of fast computation. It consists of estimating the dimension of a time varying signal (waveform) directly in the time domain, which allows significant savings in program run-time. The phase space representation of a nonlinear system usually describes an attractor with fractional dimension. This attractor dimension is invariant, even under different initial conditions. This explains why the FD of this attractor has been used widely for system characterization. However, estimating the FD of this attractor involves a large computational burden. An embedding system has to be constructed from the original time-domain signal, based on the method of delays, and the attractor of this system has to be unfolded before estimating its FD. At present, the algorithms developed to assess the FD of the attractor are very slow, due to a considerable requirement for preprocessing. The most popular method for doing this is the algorithm from Grassberger and Proccacia [6], which estimates the correlation dimension (D_2) or FD of the attractor. Many other algorithms for estimating the FD of the attractor have been proposed, but their computational requirements are prohibitive. Three of the most prominent methods for computing the FD of a waveform [1], [2], [5] have been applied to the analysis of EEG, other biomedical signals, and a variety of engineering systems. Though our study focuses on experimental signals derived from IEEG, its results are widely applicable to any type of signal.

FD ALGORITHMS ANALYZED

Higuchi's Algorithm

Consider $x(1), x(2), \cdots, x(N)$ the time sequence to be analyzed. Construct k new time series x_m^k as:

$$x_m^k = \left\{ x(m), x(m+k), x(m+2k), \cdots, x\left(m+\left\lfloor \frac{N-m}{k} \right\rfloor k\right), \right\}$$

for m=1, 2, ..., k, where m indicates the initial time value, and k indicates the discrete time interval between points, and $\lfloor a \rfloor$ means integer part of a. For each of the k time series or curves x_m^k, the length $L_m(k)$ is computed by:

$$L_m(k) = \frac{\sum_{i=1}^{\left\lfloor \frac{N-m}{k} \right\rfloor} \left| x(m+ik) - x(m+(i-1)k) \right| (n-1)}{\left\lfloor \frac{N-m}{k} \right\rfloor k} \quad (1)$$

where N is the total length of the data sequence x and $(N-1) / \left\lfloor \frac{N-m}{k} \right\rfloor k$ is a normalization factor. An average length is computed as the mean of the k lengths $L_m(k)$ for m=1,...,k This procedure is repeated for each k ranging from 1 to kmax, obtaining an average length for each k. In the curve of ln(L(k)) versus ln(1/k), the slope of the least-squares linear best fit, is the estimate of the fractal dimension [1].

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Petrosian's Algorithm

Petrosian uses a quick estimate of the FD [5]. However this estimate is really the FD of a digital sequence as defined by Katz [2]. Since waveforms are analog signals, a digital signal is derived by subtracting consecutive samples from the waveform record. From this sequence of subtractions, a binary sequence is created assigning +1 or −1 if the result of the subtraction is positive or negative respectively. The FD is then computed as:

$$D = \frac{\log_{10} n}{\log_{10} n + \log_{10}\left(\frac{n}{n + 0.4 N_\Delta}\right)} \qquad (2)$$

where n is the length of the sequence (number of points), and N_Δ is the number of sign changes (number of dissimilar pairs) in the binary sequence generated.

Katz's Algorithm

In contrast to Petrosian's method, Katz's FD calculation [2] is slightly slower, but it is derived directly from the waveform, eliminating the preprocessing step. The FD of a curve can be defined as:

$$D = \frac{\log_{10}(L)}{\log_{10}(d)} \qquad (3)$$

where L is the total length of the curve or sum of distances between successive points, and d is the diameter estimated as the distance between the first point of the sequence and the point of the sequence that provides the farthest distance. Mathematically speaking, d can be expressed as:

$$d = \max(\text{distance}(1,i)) \qquad (4)$$

Considering the distance between each point of the sequence and the first, point i is the one that maximizes the distance with respect to the first point.

The FD compares the actual number of units that compose a curve with the minimum number of units required to reproduce a pattern of the same spatial extent. FDs computed in this fashion depend upon the measurement units used. If the units are different, then so are the FDs. Katz's approach solves this problem by creating a general unit or yardstick: the average step or average distance between successive points, $\underline{a}$. Normalizing distances in Equation (3) by this average results in:

$$D = \frac{\log_{10}(L/\underline{a})}{\log_{10}(d/\underline{a})} \qquad (5)$$

Defining n as the number of steps in the curve, then $n = L/\underline{a}$, and (5) can be written as:

$$D = \frac{\log_{10}(n)}{\log_{10}\left(\frac{d}{L}\right) + \log_{10}(n)} \qquad (6)$$

Expression (6) summarizes Katz's approach to calculate the FD of a waveform.

METHODS

We tested these algorithms with respect to reliability, efficiency, noise sensitivity, and record length. Each of the algorithms described above was implemented in Matlab, and tested on synthetic signals with known FD, and on experimental data derived from EEG signals of epileptic patients.

Synthetic data were produced using the deterministic Weierstrass cosine function [7], given as follows:

$$W_H(t) = \sum_{i=0} \gamma^{-iH} \cos(2\pi\gamma^i t), \qquad 0 < H < 1 \qquad (7)$$

where $\gamma > 1$. The fractal dimension of this signal is given by $D = 2 - H$. A set of one hundred sequences, each with different FD, was generated using (7).

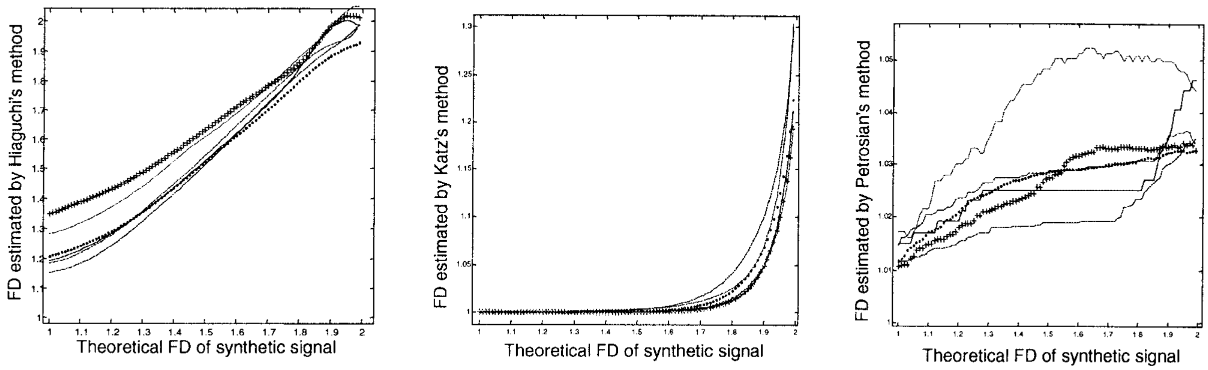
The FD of the experimental signals was computed using a sliding window. A total of 16 seizure records were analyzed. As the sliding window moved over the data, the FD was computed for each set of points that lay inside the window. A sliding window of 1.25 s was used to assure stationarity in the segment analyzed, considering that our EEGs were sampled at 200Hz.

RESULTS

FDs of synthetic signals ranged from 1.001 to 1.991. Figures 1.a,b,c show the FD values obtained by each of the analysis methods plotted against the known FDs of the synthetic data. Note that perfect reproduction of the known FDs should yield a straight line of slope equal one. Hiaguchi's algorithm (Figure 1.a) provides the most accurate estimates of the FD. Katz's method (Fig. 1.b) is less linear. Its calculated FDs were exponentially related to the known FDs. Petrosian's method (Fig. 1.c) is relatively linear and demonstrated the least dynamic range for the estimated FD (approximately between 1.01 and 1.055). The FD estimates with Hiaguchi's method improve as the window length increases. No window length effect is observed in the range of 150 to 2000 points for Petrosian's method. In Katz's method the window length affects the dynamic range of the estimated FD with respect to the true FD. The FD results obtained with experimental EEG data, reveal that even though Hiaguchi's method is the most accurate of the three, Katz's method yielded the most consistent results regarding discrimination between states of brain function. Specifically, when considering the distinction between the period before an epileptic seizure (preictal period) and the seizure period (ictal period), Katz's technique provided the most repeatable and discriminative results between preictal (pre-seizure) and ictal (seizure) phases over 16 EEG records analyzed [8].

Figure 2.a-c presents as example four records analyzed from one epileptic patient by the three FD methods introduced earlier. Equivalent results were obtained for all the records studied. Time labeled as zero corresponds to the beginning of the ictal period. The better performance of Katz's algorithm over Hiaguchi's can be explained by the exponential characteristic observed in Fig. 1.b. Because of its exponential relationship to known FDs, Katz's method emphasizes the higher FDs, which

presumably contribute the most to discriminating different states of brain function with respect to seizures. In these cases, it appears that the true value of the FD is not as important as the changes in FD associated with different brain states, a feature that may be desirable in other systems.



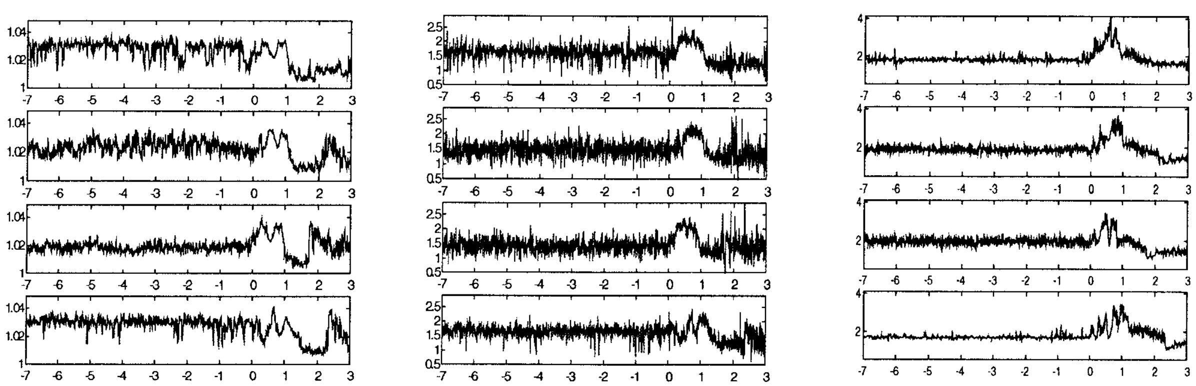
(a) (b) (c)

Figure 1: FD by each method versus theoretical FD of synthetic signal for different number of points (N).
(a) By hiaguchi's method, +++ N=150, --- N=250, **...** N=500, — N=750, -.- N=1000, −−− N=2000;
(b) By Katz's method, -.-.- N=150, −−− N=250, **...** N=500, — N=750, --- N=1000, +++ N=2000;
(c) By Petrosian's method, +++ N=150, --- N=250, **...** N=500, — N=750, -.- N=1000, −−− N=2000

Figure 3.a-c presents the FD estimated for each method when the synthetic signal is contaminated with white noise, yielding a signal to noise ratio (SNR) of 10db. Hiaguchi's and Petrosian's methods (Fig. 3.a,c) are severely affected by this level of noise. However, Katz method is influenced by the noise, but not as much as the other methods. Analyses for different SNRs demonstrate that Hiaguchi's and Petrosian's methods deteriorate for low SNRs and that Katz' algorithm is the most immune to the noise effects.

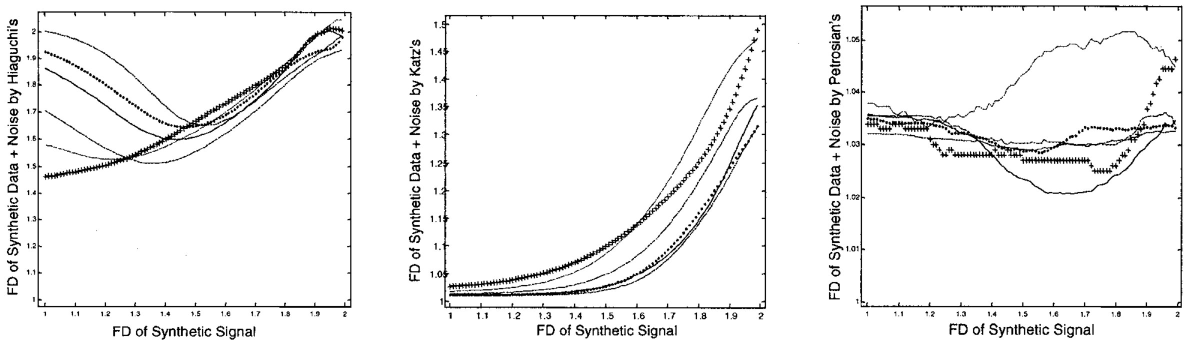
A comparison of computational burden between the methods for different window lengths is presented on Table 1. Katz method has the highest number of floating point operations (flops), around double the flops of Hiaguchi's, however it is computationally faster for small data records. For records of 2000 points length or more run-times for Katz's and Hiaguchi's are in the same order.

Katz's is 3.25% slower than the fastest method (Petrosian's), and Hiaguchi's is 19.25% when considering a window length of 500 points. However, this is not an issue, since the total record length is 12min, therefore, all methods can be run in real time. If the window length increases up to 8000 points, then Hiaguchi's performance improves, and is only 13% slower than Petrosian's.



(a) (b) (c)

Figure 2: FD of EEG signals for (a) Petrosian's method (b) Hiaguchi's method, and (c) Katz's method



(a) (b) (c)

Figure 3: Effect of noise on the FD estimate (SNR=10db) by (a) Hiaguchi's; (b) Katz's; and (c) Petrosian's method.

Table 1: Comparison of computational burden and run-time

Window Length	Flops			Run-time		
	Hiaguchi's	Petrosian's	Katz'	Hiaguchi's	Petrosian's	Katz'
250	2566403	1625603	4028803	28.78	8.9	9.17
500	2261173	1610787	4009385	15.27	5.22	5.39
1000	2101443	1598371	3987167	9.39	5.16	5.38
2000	2015247	1587171	3963567	5.76	4.12	4.29
4000	Note: Records analyzed were 12min length			4.28	3.3	3.57
8000	sampled at 200Hz with 36% overlap			3.24	2.85	3.24

With respect to the fastest method (Petrosian's), both algorithms are slower, Katz's by 3.25% and Hiaguchi's by 19.25%, when using a window length of 500 points. This is not a problem for the total record length of 12min, since all three methods can be run in real time. If the window length increases up to 8000 points, then Hiaguchi's performance improves, and is only 13% slower than Petrosian's.

CONCLUSION

Our results show that Katz's algorithm is the most consistent method for discrimination of epileptic states from the IEEG, likely due to its exponential transformation of FD values and relative insensitivity to noise. Hiaguchi's method, however, yields a more accurate estimation of signal FD, when tested on synthetic data, but is more sensitive to noise. Petrosian's method may be less suitable for analog signal analysis, given its poor reproducibility of dynamic range of synthetic FDs. This study demonstrates that a careful selection of FD algorithm is required for specific applications. Factors such as knowledge of possible FD range, noise level, and window length must be considered to achieve the best results.

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