XIX Coloquio Latinoamericano de Álgebra
December 2012, Pucón, Chile

Special $\star$-Identities in group algebras and oriented group involutions

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Abstract

Let $R^+ = \{r \in R : r^\star = r\}$ and $R^- = \{r \in R : r^\star = -r\}$ be the set of *symmetric and skew-symmetric elements* respectively of the ring R under the involution $\star$. A question of general interest is to determine the extent to which the properties of A^+ or A^- determine the properties of the whole algebra A. One of the most famous and lovely results in this direction is the following theorem due to Amitsur (see [5, Theorem 6.5.1]):

"*If R is a ring with involution $\star$ and R satisfies a $\star$-P.I, then R satisfies a P.I. In particular, if R^+ or R^- is **P.I**, then R is **P.I***".

Given both a nontrivial homomorphism $\sigma : G \to \{\pm 1\}$ (called an **orientation**) and an involution $^* : G \to G$ extended linearly to the group algebra $\mathbb{F}G$, an *oriented group involution* of $\mathbb{F}G$ is defined by

$$\alpha = \sum_{g \in G} \alpha_g g \mapsto \alpha^\star = \sum_{g \in G} \alpha_g \sigma(g) g^*.$$

Notice that, as σ is nontrivial, $char(R)$ must be different from 2. It is clear that, $\alpha \mapsto \alpha^\star$ is an involution of $\mathbb{F}G$ if and only if $gg^* \in N = ker(\sigma) = \{g \in G : \sigma(g) = 1\}$ for all $g \in G$. In the case that the involution on G is the classical involution, $g \mapsto g^{-1}$, the map $\star$ is precisely the *oriented classical involution* introduced by S. P. Novikov (1970) in the context of K-theory (see [1]).

In this talk we present some new results about group algebras such that either $\mathbb{F}G^+$ or $\mathbb{F}G^-$ are Lie nilpotent, i.e., such that for all $z_i \in \mathbb{F}G^+$ (or for all $w_i \in \mathbb{F}G^-$) the *special $\star$-identity* $[z_1 + z_1^\star, z_2 + z_2^\star, ..., z_n + z_n^\star] = 0$ (or $[w_1 - w_1^\star, w_2 - w_2^\star, ..., w_n - w_n^\star] = 0$), is satisfied for some $n \geq 2$ or are Lie m-Engel, i.e., for $m \in \mathbb{N}$, $[\alpha + \alpha^\star, \underbrace{\beta + \beta^\star, ..., \beta + \beta^\star}_{m \text{ times}}] =$

*Partially supported by **CAPES** - Brazil and **C**omissão **C**oordenadora de **P**rograma, IME-USP, São Paulo, Brazil, e-mail: `aholguin@ime.usp.br`

†Joint work with César Polcino Milies IME-USP, CMCC-UFABC Brasil and John H. Castillo DME-UDENAR Colombia, IME-USP.

0 (or $[\alpha-\alpha^\star, \underbrace{\beta-\beta^\star,...,\beta-\beta^\star}_{m \text{ times}}]=0$) is a P.I for all $\alpha,\beta\in\mathbb{F}G^+$ (or $\alpha,\beta\in\mathbb{F}G^-$). These results, represent a continuation of the work of Castillo, Giambruno, Polcino Milies and Sehgal, see [1], [2], [3] and [4]. For instance we show:

Let $\zeta(G)$ be the center of G and suppose that $\tilde{\zeta}(G)=\{z^{-1}z^* : z\in\zeta(G)\}$ is a infinite. Then, $\mathbb{F}G^+$ or $\mathbb{F}G^-$ is Lie nilpotent of index n if and only if $\mathbb{F}G$ is Lie nilpotent of index n.

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