Oriented group involutions in group algebras

Alexander Holguín Villa *
Escuela de Matemáticas - UIS †
Bucaramanga - Colombia

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Abstract

Let $\mathbb{F}G$ denote the group algebra of the group G over the field $\mathbb{F}$ with $char(\mathbb{F}) \neq 2$, and let $\circledast : \mathbb{F}G \to \mathbb{F}G$ denote the involution defined by $\alpha = \Sigma_{g\in G}\alpha_g g \mapsto \alpha^{\circledast} = \Sigma \alpha_g \sigma(g) g^*$, where $\sigma : G \to \{\pm 1\}$ is a group homomorphism (called an orientation) and $*$ is an involution of G.

We write $\mathbb{F}G^+ = \{\alpha \in \mathbb{F}G : \alpha^{\circledast} = \alpha\}$ and $\mathbb{F}G^- = \{\alpha \in \mathbb{F}G : \alpha^{\circledast} = -\alpha\}$ for the set of symmetric and skew-symmetric elements of $\mathbb{F}G$ under $\circledast$. Let $\mathcal{U}^+(\mathbb{F}G) = \mathcal{U}(\mathbb{F}G) \cap \mathbb{F}G^+$ denote the set of $\circledast$-symmetric units.

In this talk we present some new results about group algebras such that either $\mathbb{F}G^+$ or $\mathbb{F}G^-$ are Lie nilpotent (Lie n-Engel). Furthermore we study $\circledast$-group identities in the set $\mathcal{U}^+(\mathbb{F}G)$ and we prove, under some assumptions, that if the set of $\circledast$-symmetric units of $\mathbb{F}G$ satisfies a group identity then $\mathbb{F}G$ satisfy a polynomial identity, i.e., we obtain an affirmative answer to Hartley's Conjecture in this setting. Moreover, in case when the prime radical $\eta(\mathbb{F}G)$ of $\mathbb{F}G$ is nilpotent we characterize the groups for which the symmetric units $\mathcal{U}^+(\mathbb{F}G)$ do satisfy a group identity, [3].

References

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*e-mail: aholguin@uis.edu.co; alexholguinvilla@gmail.com
†**UIS:** Universidad Industrial de Santander