# Lie properties of symmetric elements in regard to an oriented group involution


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**Abstract**

Let $\mathbb{F}G$ denote the group algebra of the group $G$ over the field $\mathbb{F}$ with $char(\mathbb{F}) \neq 2$, and let $\dagger : \mathbb{F}G \to \mathbb{F}G$ denote the involution defined by $\alpha = \Sigma\alpha_g g \mapsto \alpha^\dagger = \Sigma\alpha_g\sigma(g)g^*$, where $\sigma : G \to \{\pm 1\}$ is a group homomorphism (called an orientation morphism) and $*$ is an involution of the group $G$.

We denote $\mathbb{F}G^+ = \{\alpha \in \mathbb{F}G : \alpha^\dagger = \alpha\}$ and $\mathbb{F}G^- = \{\alpha \in \mathbb{F}G : \alpha^\dagger = -\alpha\}$ the set of symmetric and skew-symmetric elements of $\mathbb{F}G$ under $\dagger$.

In an associative ring $R$, we define the Lie product via $[x_1, x_2] = x_1x_2 - x_2x_1$ and, we can extended this recursively via

$$[x_1, ..., x_n, x_{n+1}] = [[x_1, ..., x_n], x_{n+1}].$$

Let $S$ be a subset of R. We say that $S$ is *Lie nilpotent* if there exists an $n \geq 2$ such that $[a_1, ..., a_n] = 0$ for all $a_i \in S$. The smallest such $n$ is called index of nilpotency of $S$. For a positive integer $n$, we say that $S$ is Lie $n$-Engel if

$$[a, \underbrace{b, ..., b}_{n \text{ times}}] = 0$$

for all $a, b \in S$. Obviously if $S$ is Lie nilpotent then it is Lie $n$-Engel for some $n$.

Lie nilpotent (Lie $n$-Engel) group algebras have been the subject of a good deal of attention; indeed, it is interesting to know the extent to which the Lie properties of the *symmetric* (*or skew-symmetric*) *elements* determine the Lie properties of the whole group algebra.

In this poster we present some results about group algebras such that either $\mathbb{F}G^+$ or $\mathbb{F}G^-$ are Lie nilpotent (Lie $n$-Engel). For instance we show:

*Let $\zeta(G)$ be the center of $G$ and suppose that $\tilde{\zeta}(G) = \{z^{-1}z^* : z \in \zeta(G)\}$ is infinite. Then, $\mathbb{F}G^+$ or $\mathbb{F}G^-$ is Lie nilpotent of index $n$ if and only if $\mathbb{F}G$ is Lie nilpotent of index $n$.*



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