Normal group algebras and oriented group involutions

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Groups, Rings and Group Rings 2012
Ubatuba - São Paulo, Brazil

July 23-28, 2012

Abstract

Let RG denote the group algebra of a group G over a commutative ring R with unity. Given both a nontrivial homomorphism $\sigma : G \to \{\pm 1\}$ (called an orientation) and an involution $* : G \to G$ extended linearly to the group algebra RG, an oriented involution of RG is defined by

$$\alpha = \sum_{g \in G} \alpha_g g \mapsto \alpha^{\dagger} = \sum_{g \in G} \alpha_g \sigma(g) g^*.$$

Notice that, as σ is nontrivial, $char(R) \neq 2$. It is clear that $\alpha \mapsto \alpha^{\dagger}$ is an involution of RG if and only if $gg^* \in ker(\sigma) = \{g \in G : \sigma(g) = 1\} = N$ for all $g \in G$.
A ring R with involution $*$ is said to be *normal* if and only if $rr^* = r^*r$, for all $r \in R$.

Let $\zeta(G) = \zeta$ denote the center of the group G and recall that G is called LC-group if it is nonabelian and for every pair of elements $g, h \in G$, we have that $gh = hg$ if and only if either $g \in \zeta$, or $h \in \zeta$, or $gh \in \zeta$. A group is SLC if it is LC and has a unique nontrivial commutator.

Using unpublished results of Felzenszwab, Giambruno, Leal and Polcino, we characterize group algebras RG which are normal in regard to an oriented group involution. The results depend on whether N is either abelian or an SLC-group.

*The author's research (Ph.D. Student, IME-USP) has been partially supported by **Capes** - Brazil.
†Ph.D.'s Advisor: Professor César Polcino Milies.