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Group identities in group algebras and oriented group involutions

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Abstract

Let $\mathbb{F}G$ denote the group algebra of the locally finite group G over the infinite field $\mathbb{F}$ with $char(\mathbb{F}) \neq 2$, and let $\circledast : \mathbb{F}G \to \mathbb{F}G$ denote the involution defined by $\alpha = \Sigma_{g \in G} \alpha_g g \mapsto \alpha^{\circledast} = \Sigma \alpha_g \sigma(g) g^*$, where $\sigma : G \to \{\pm 1\}$ is a group homomorphism (called an orientation) and $*$ is an involution of G.

We write $\mathbb{F}G^+ = \{\alpha \in \mathbb{F}G : \alpha^{\circledast} = \alpha\}$ and $\mathbb{F}G^- = \{\alpha \in \mathbb{F}G : \alpha^{\circledast} = -\alpha\}$ for the set of symmetric and skew-symmetric elements of $\mathbb{F}G$ under $\circledast$. Let $\mathcal{U}^+(\mathbb{F}G) = \mathcal{U}(\mathbb{F}G) \cap \mathbb{F}G^+$ denote the set of $\circledast$-symmetric units.

Some time ago, Brian Hartley made the following conjecture: Let G be a torsion group and $\mathbb{F}$ an infinite field. If $\mathcal{U}(\mathbb{F}G)$ satisfies a group identity, then $\mathbb{F}G$ satisfies a polynomial identity, [6, 7, 3, 1, 2].

We prove, under some assumptions, that if the set of $\circledast$-symmetric units of $\mathbb{F}G$ satisfies a group identity then $\mathbb{F}G$ satisfy a polynomial identity, i.e., we obtain an affirmative answer to Hartley's Conjecture in this setting. Moreover, in case when the prime radical $\eta(\mathbb{F}G)$ of $\mathbb{F}G$ is nilpotent we characterize the groups for which the symmetric units $\mathcal{U}^+(\mathbb{F}G)$ do satisfy a group identity, [4, 5].

References

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